Assessment of the stability of an optimized cooling configuration of Gyrotron resonant cavity for nuclear fusion machines through an analytical model

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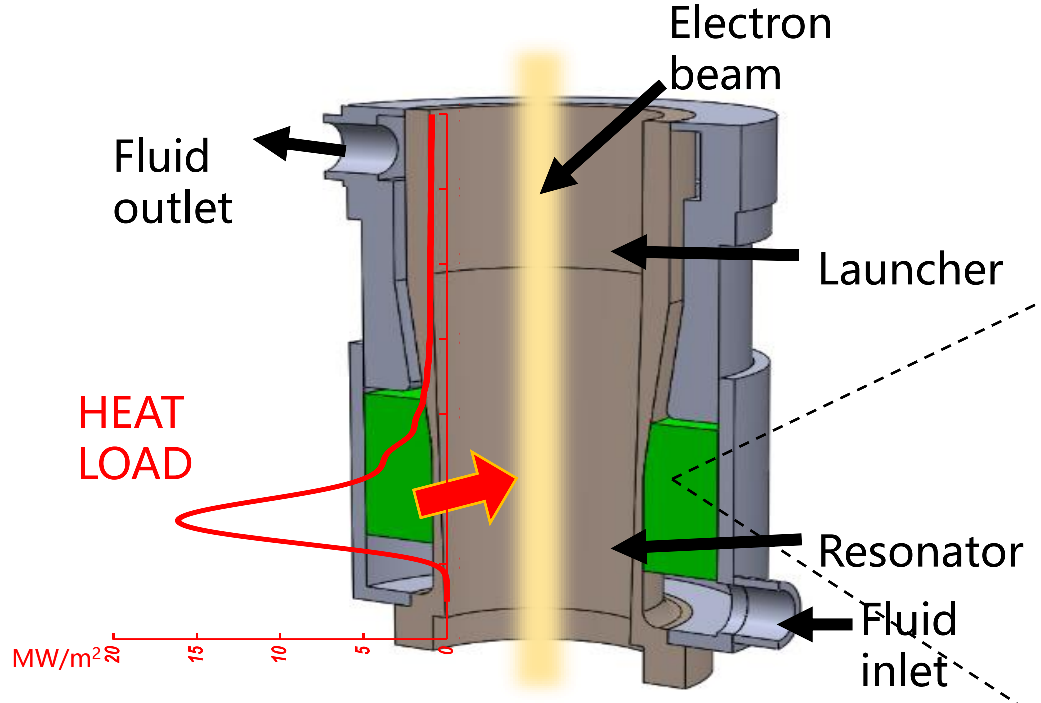
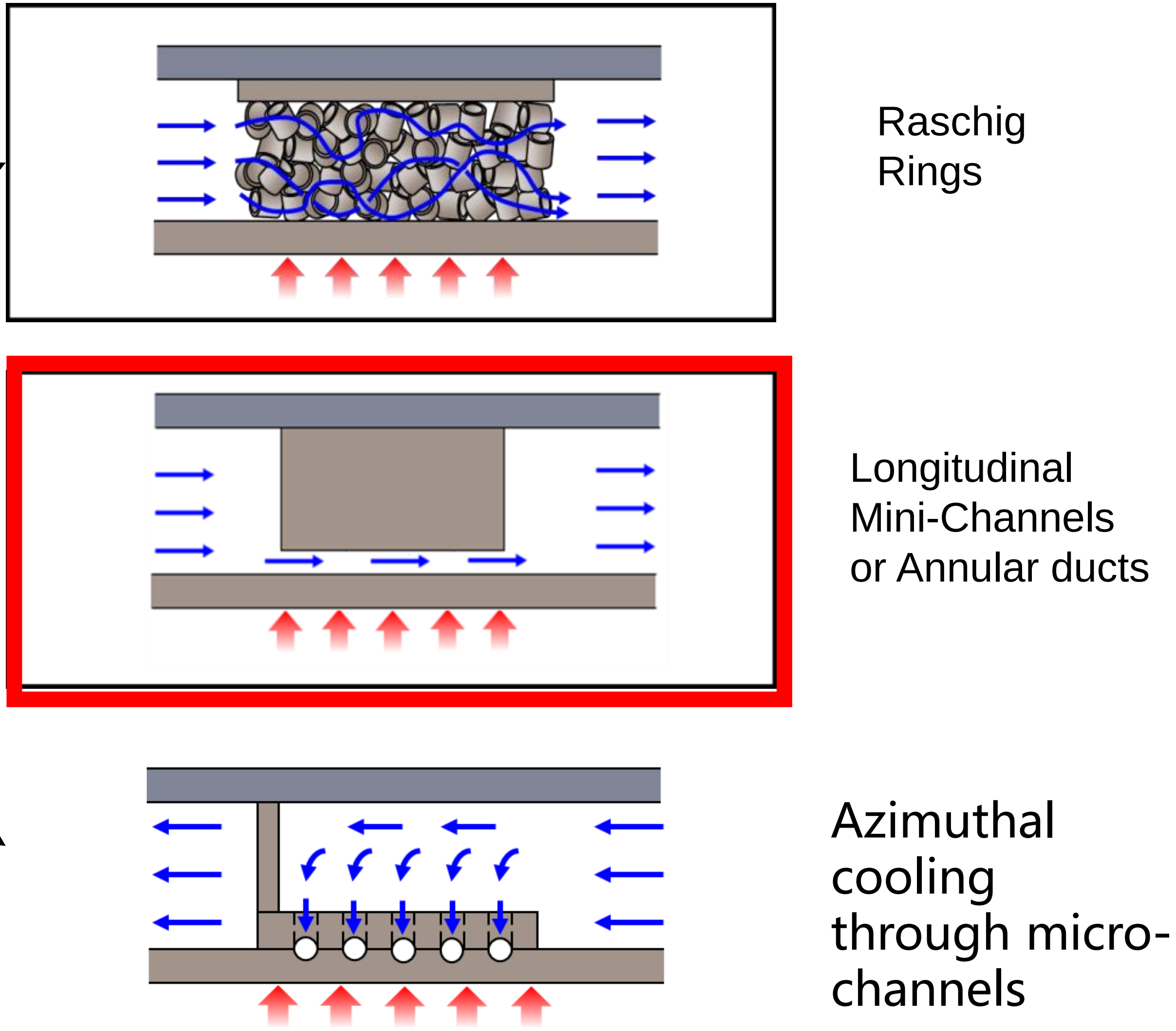
Outlines

- Introduction on Gyrotron cavity cooling
- Optimization strategies adopted
- Aim of the study
- Annular geometry linear model and results
- Comparison with non linear step response
- Conclusions and future perspectives

Operation of the gyrotron cavity: cooling strategies

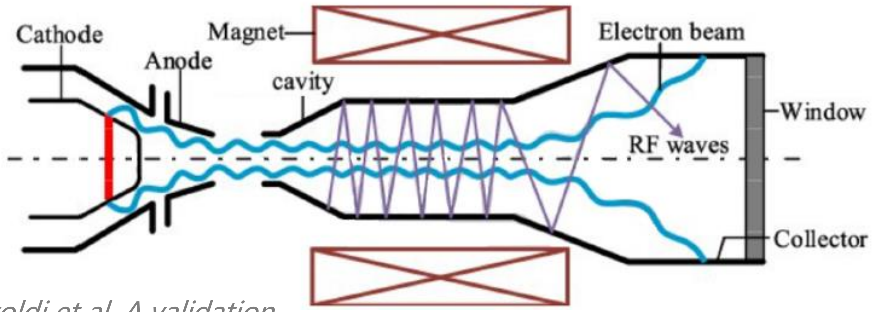




- Gyrotrons are a candidate technology for the plasma external heating of nuclear fusion machines.
- Its performance crucially depends on the cavity heat sink capability

Operation of the gyrotron cavity: a multi-physics problem

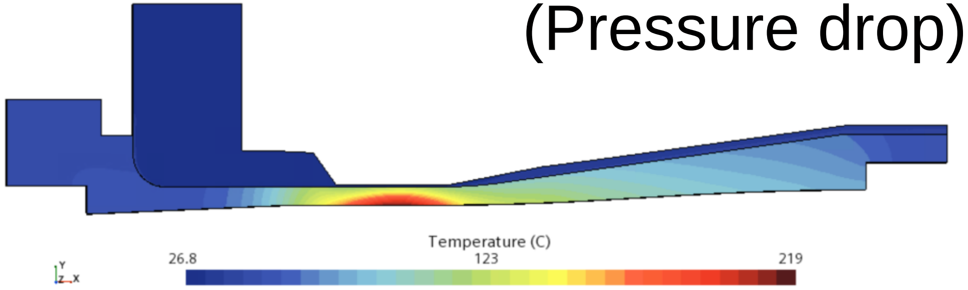
Frequency shift
(driving the heat load)

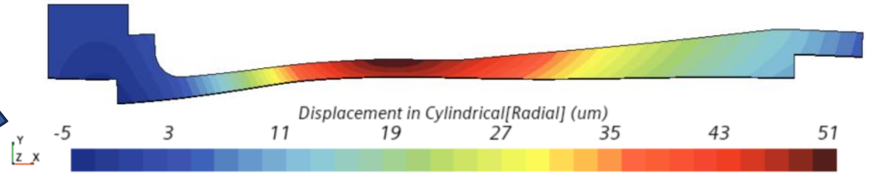


[L Savoldi et al, A validation roadmap for Multiphysics simulators for resonator of MW-Class of Gyrotron for fusion applications]

Heat Load → high Temperature on inner cavity wall

(Pressure drop)



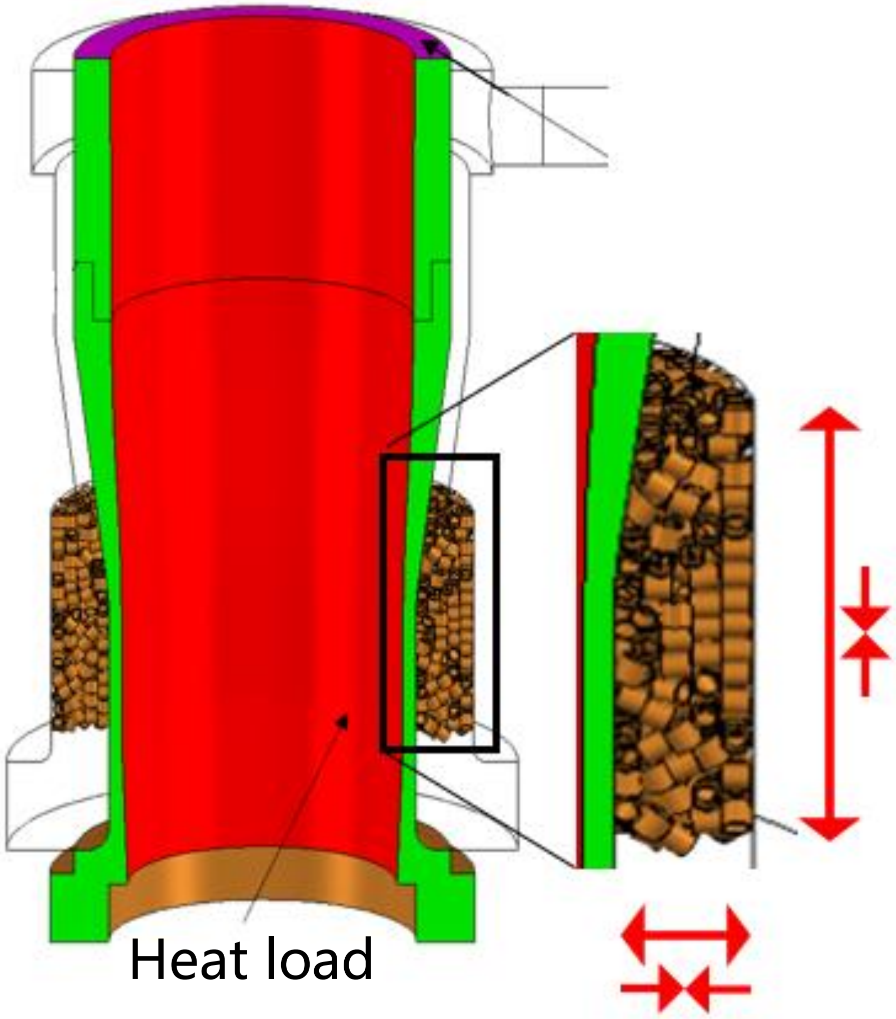


Thermal deformation

(Stresses)

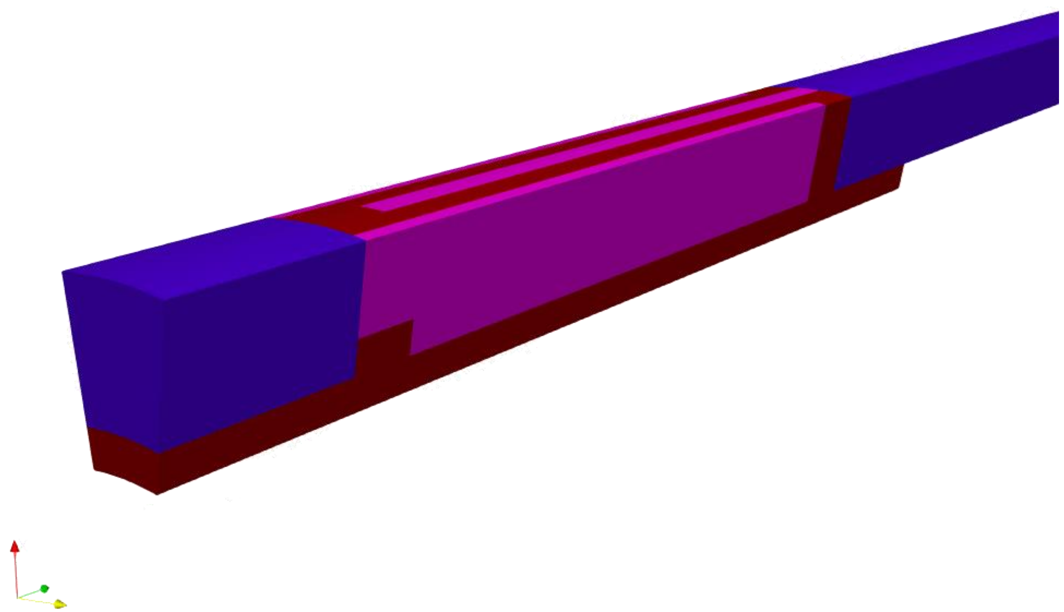
Optimization needed!

Improvement for RR cooling



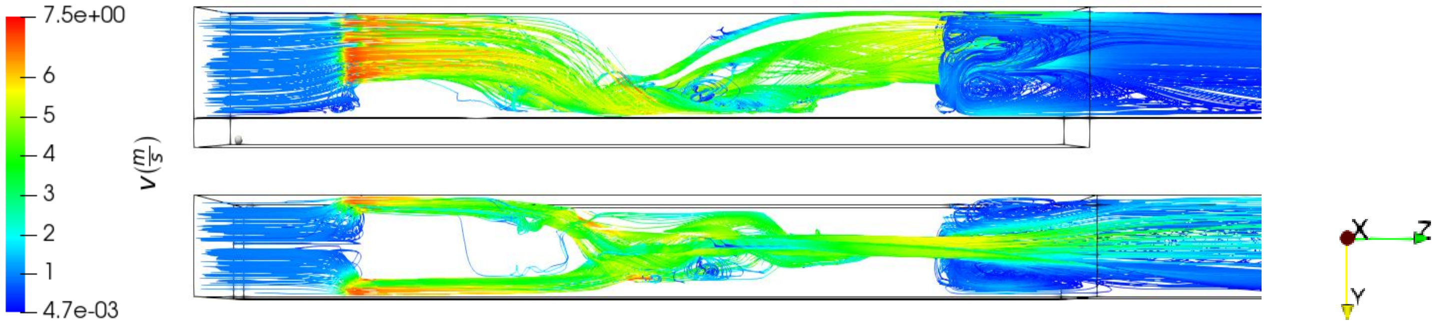
- Cooling using Raschig Ring matrix adopted for W7-X and foreseen for the European gyrotrons for ITER, DTT, TCV
- "Optimization" performed by trial and error, stretching or shrinking the area equipped with RR
- Resulting improved cooling already implemented in, e.g., dual frequency TCV gyrotrons

Optimization for azimuthal cooling



- For azimuthal cooling, adjoint topology optimization performed on the region of the manifolds and of the azimuthal flow

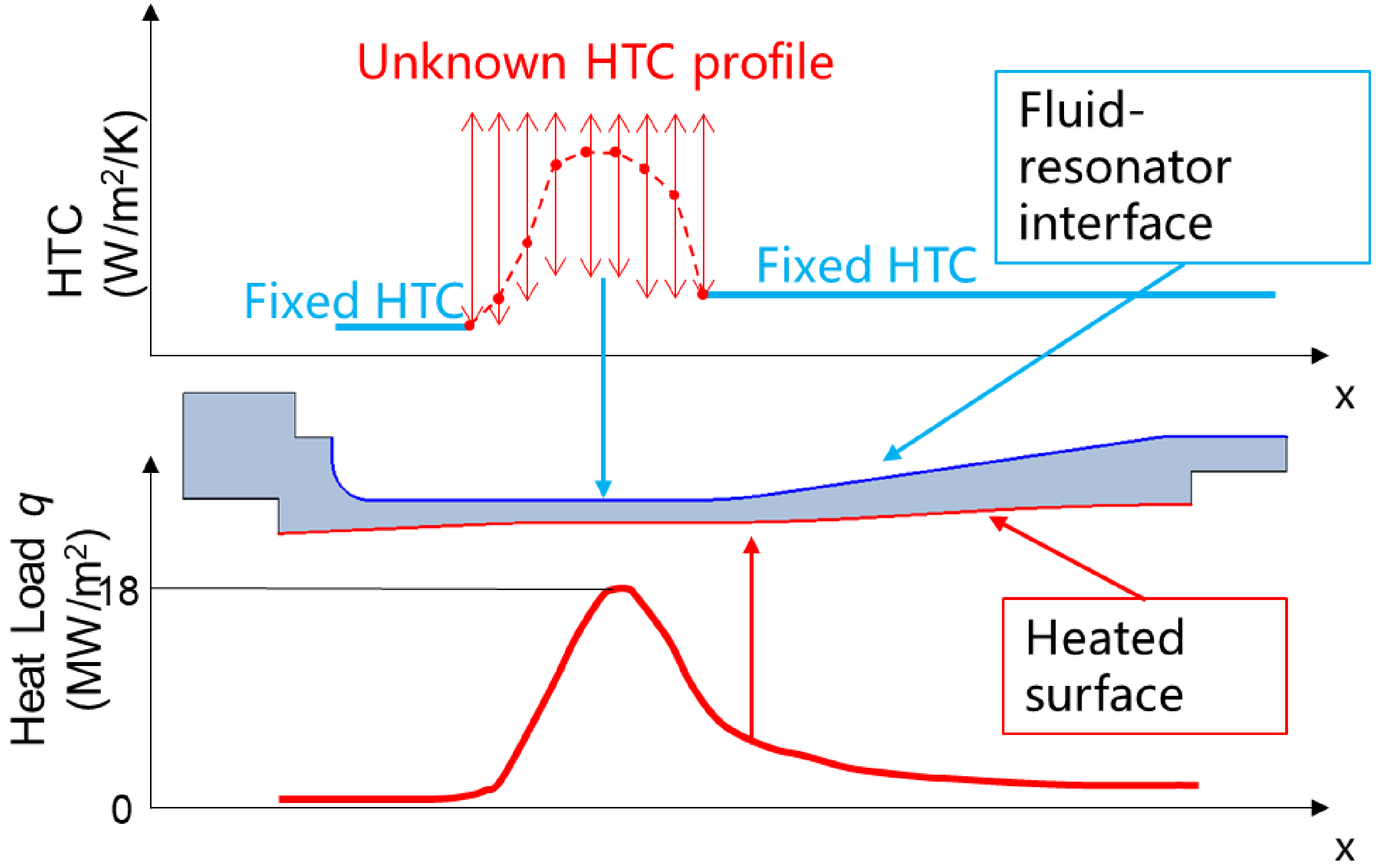
- Cost function weighting the mean temperature T_{mean} and its standard deviation $stdev(T)$, with constraint on the pressure drop



- Results are encouraging, but deformation (and stress) are not controlled directly

Biogeography-Based Optimization for longitudinal cooling (I)

[R. Difonzo et al, IEEE Trans. On Plasma Science 2022]



Cost function C

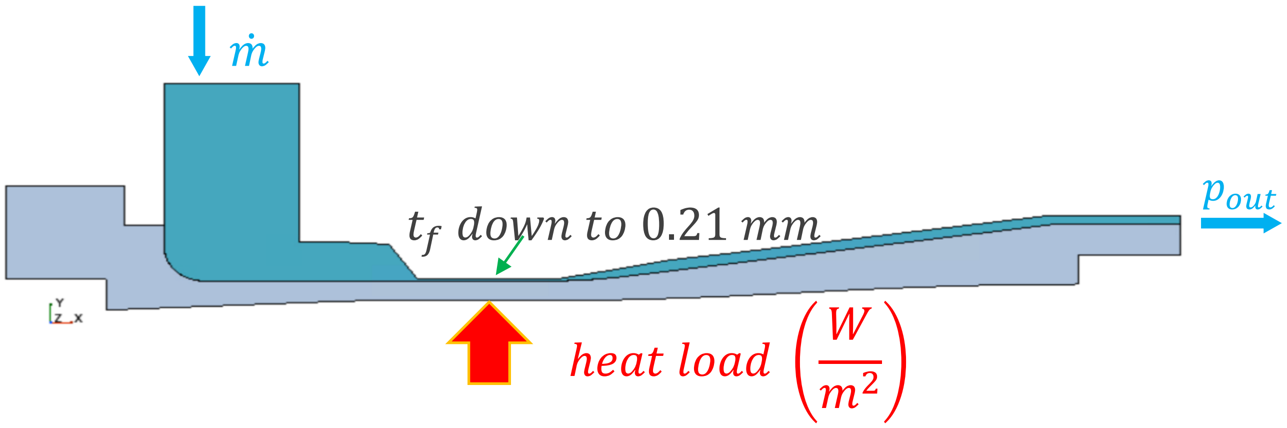
$$C = \frac{\int_{x_1}^{x_2} disp(x) * q(x)dx}{\int_{x_1}^{x_2} q(x)dx} \text{ (µm)}$$

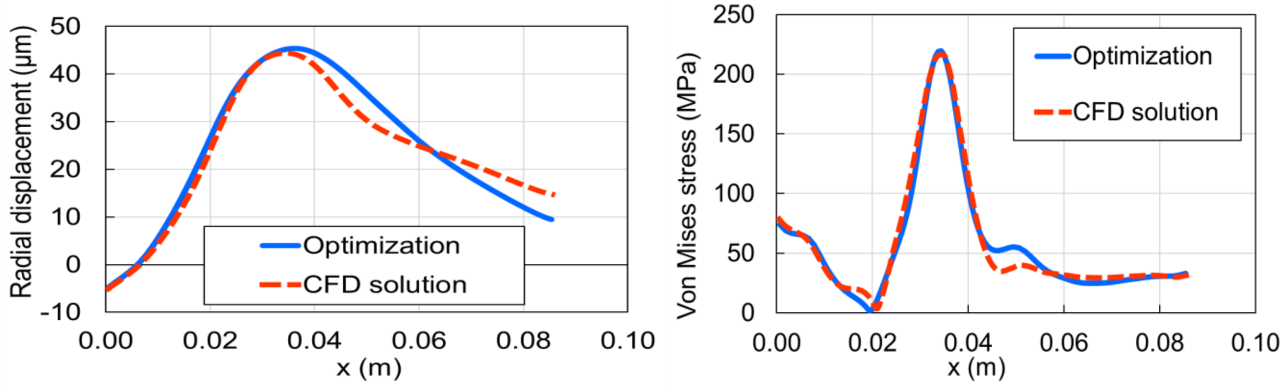
$$C' = C + \alpha \times (-\Delta\sigma)$$

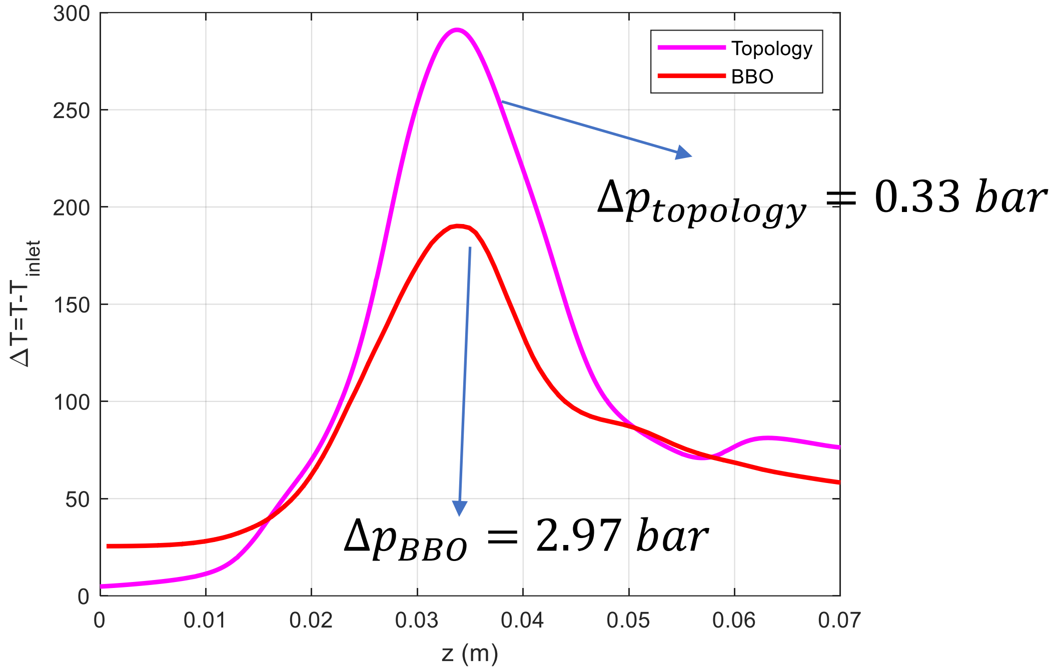
Penalty to satisfy the constraint

$$\Delta\sigma = \sigma_{yield\ strength}(T(\sigma_{max})) + -\sigma_{max} - \Delta\sigma_{security} \text{ (Pa)}$$

Biogeography-Based Optimization for longitudinal cooling (II)







[R. Difonzo et al, IEEE Trans. On Plasma Science 2022]

- Temperature & stress distribution ok BUT
- Thin annular region at the HL peak:
  - High pressure drop (we can live with it)
  - **STABILITY?**

Aim of the study

- Qualitative dynamics suggests: increase of power → variation of channel width→variation of HTC → variation of wall temperature → variation of power

- Analytical verification is missing

→HERE verify the stability of the optimized annular (longitudinal) cooling configuration by means of an analitical model

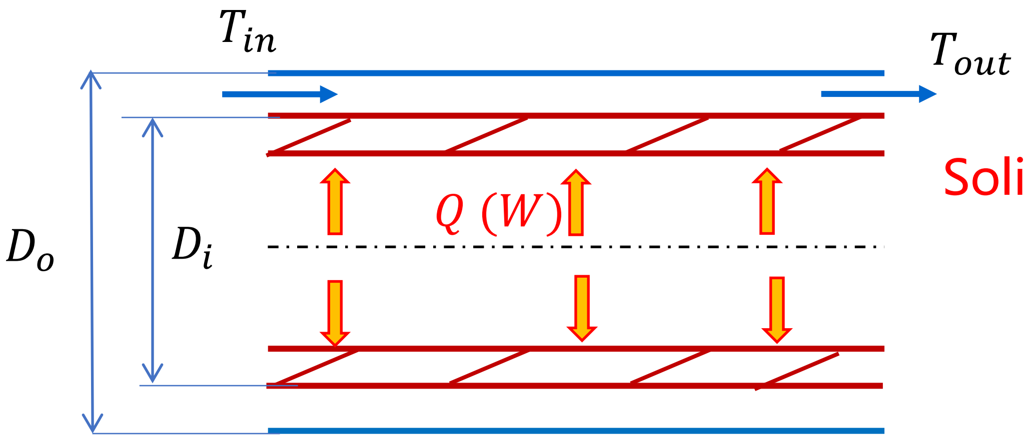
Stability

- The initial approach to the stability problem begins with *linear systems* to leverage *Lyapunov's criterion*.

- The Lyapunov criterion for the stability of linear systems in terms of *necessary and sufficient condition* on eigenvalues states: A linear system is asymptotically stable *if and only if all eigenvalues of its associated matrix have negative real parts*.

- After completing the linear stability analysis → *nonlinear system*: time-varying transients to validate whether the stability predictions, based on a linearized model, are *corroborated* by the nonlinear transients.

Analysis of the system dynamics (I)



0D thermal balance for the mean solid and coolant temperature T_m and T_c

Solid (metal)

Heat exchange surface

$$M_m C_m \frac{dT_m}{dt} = Q - hA_s(T_m - T_c)$$

Coolant

$$M_c C_c \frac{dT_c}{dt} = hA_s(T_m - T_c) - GC_c(T_{out} - T_{in})$$

$= 2T_c - 2T_{in}$

- Introducing small variations to the system dynamics:

$$M_m c_m \frac{d\delta T_m}{dt} = \delta Q - \delta hA_s \times (T_m - T_c) - hA_{heated} \times \delta(T_m - T_c)$$

$$M_c c_c \frac{d\delta T_c}{dt} = \delta hA_s \times (T_m - T_c) + hA_s \times \delta(T_m - T_c) - GC_c \times \delta(2T_c - 2T_{in})$$

Require a little bit of work…

Analysis of the system dynamics (II)

- Assumptions: $D_h = D_o - D_i$; $D_{heated} = \frac{D_o^2 - D_i^2}{D_i}$; $A_s = \pi L D_i$
- Using Dittus-Boelter correlation with $Re = \frac{4}{\pi}\frac{G}{\mu(D_o^2 - D_i^2)}$, $Nu = \frac{hD_{heated}}{k}$

$$h \times A_s = 0.023 \times k^{0.7} \times c_p^{0.3} \times \mu^{-0.5}(4G)^{0.8}\pi^{0.2}L\frac{D_i^2}{(D_o + D_i)^{\frac{9}{5}}(D_o - D_i)} =$$

$$= \gamma\frac{D_i^2}{(D_o + D_i)^{\frac{9}{5}}(D_o - D_i)} \sim \gamma D_i^{-0.8}$$

- Considering its small variation and introducing the thermal expansion law:

$$\delta D \sim \alpha D_0 \delta T_m$$

Thermal expansion coefficient

$$\delta(hA_s) = \gamma\alpha * D_i\left(\frac{2D_i}{(D_o+D_i)^{1.8}(D_o-D_i)} + \frac{-1.8D_i^2}{(D_o+D_i)^{2.8}(D_o-D_i)} + \frac{D_i^2}{(D_o+D_i)^{1.8}(D_o-D_i)^2}\right)\delta \mathrm{T_m} = \Psi\ \delta \mathrm{T_m}$$

Analysis of the system dynamics (III)

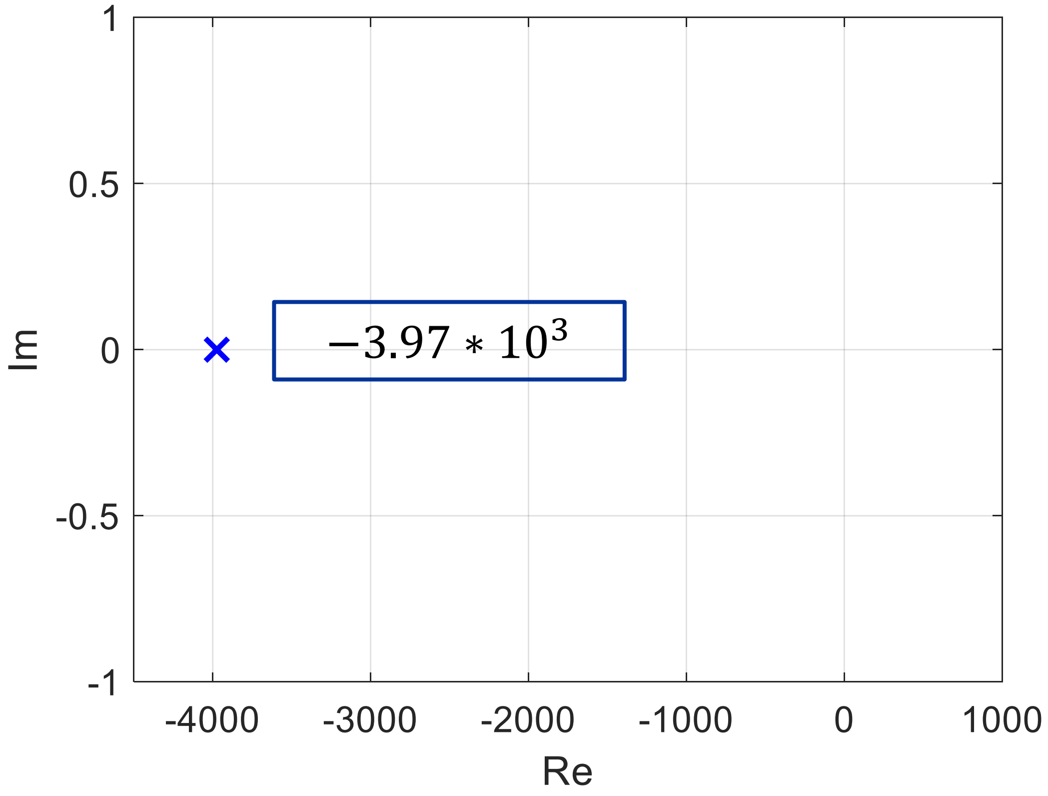
- → Final set of perturbed equations:

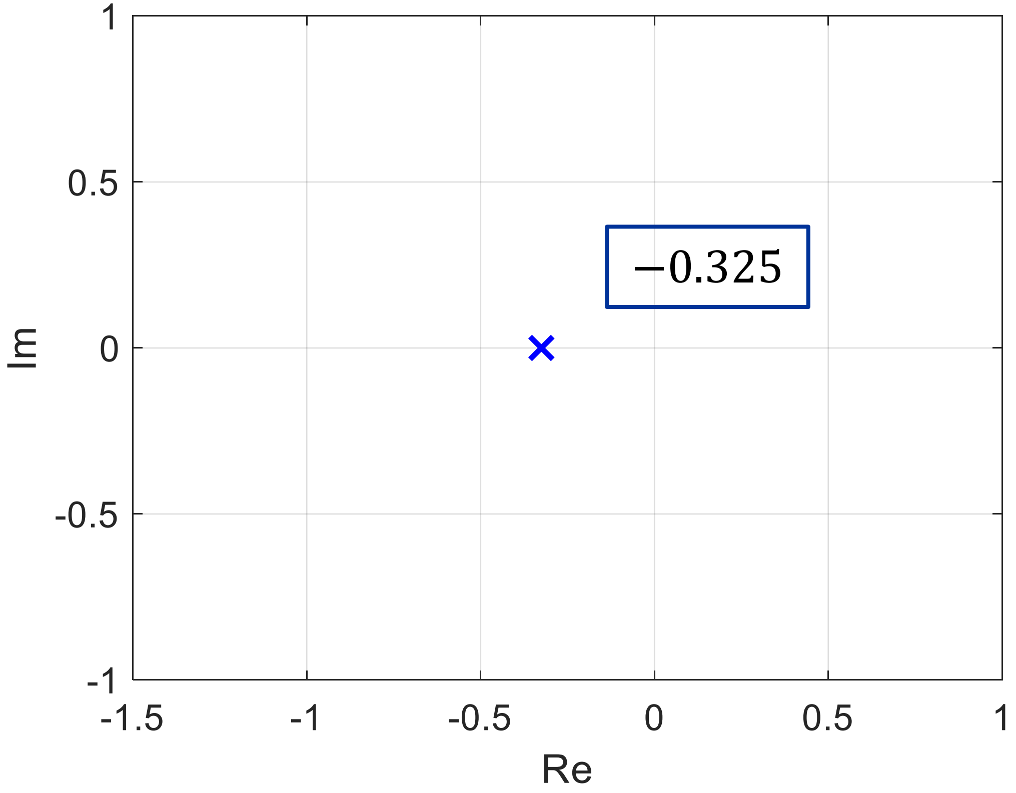
$$\frac{d}{dt}\begin{pmatrix}\delta T_m \\ \delta T_c\end{pmatrix} = \mathbf{A}\begin{pmatrix}\delta T_m \\ \delta T_c\end{pmatrix} + \mathbf{B}\begin{pmatrix}\delta Q \\ \delta T_i\end{pmatrix}$$

- Where

$$\mathbf{A} = \begin{bmatrix} \frac{-\Psi T_m^0 - hA_s + \Psi T_c^0}{M_m C_m} & \frac{\Psi T_c^0}{M_m C_m} \\ \frac{\Psi T_m^0 + hA_s - \Psi T_c^0}{M_c C_c} & 2 * \frac{G}{M_c} \end{bmatrix}$$

Linear model results





Eigenvalues present only **negative** real part

→ the **system is stable**!

Compare linear and non linear step response

Non-linear model

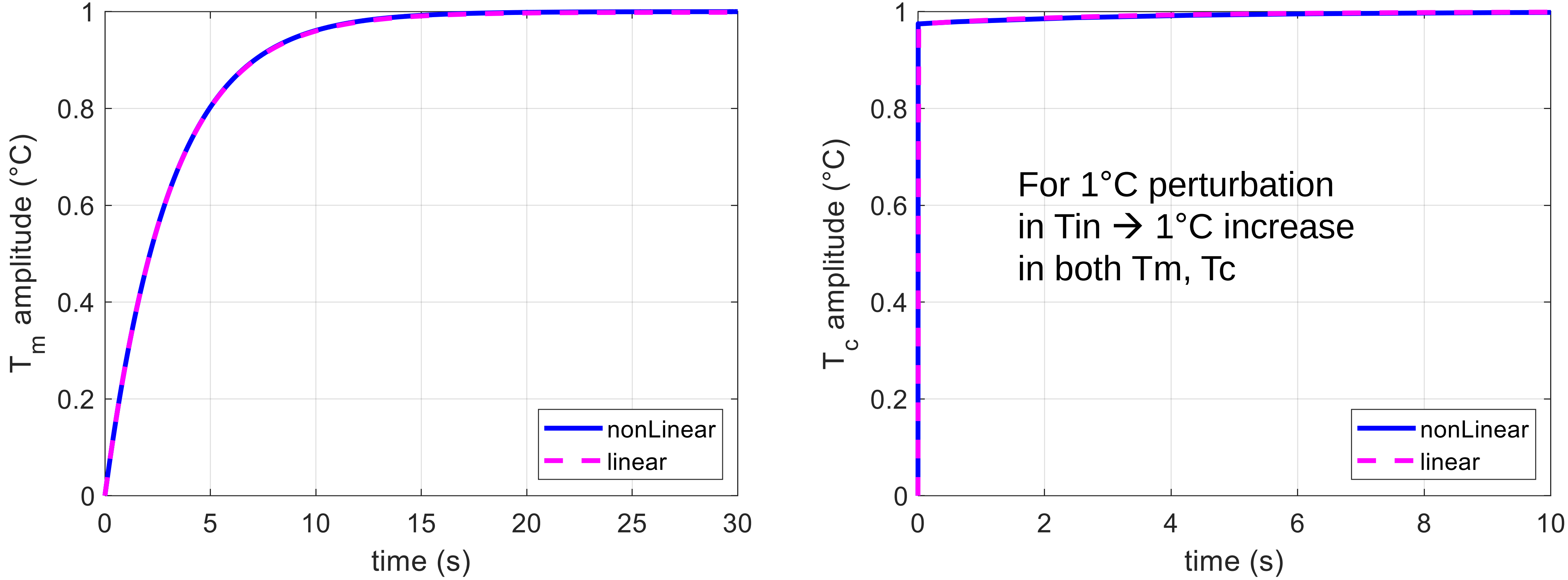
- Starting from the initial system of Ordinary Differential Equations

$$M_m C_m \frac{dT_m}{dt} = Q - hA_{heated}(T_m - T_c)$$
$$M_c C_c \frac{dT_c}{dt} = hA_{heated}(T_m - T_c) - GC_c(T_{out} - T_{in})$$

- We solve it using the ode solver in Matlab and giving perturbation in the driver separately:
  - 1 °C of perturbation in T_{in} with constant Q
  - 1 kW of perturbation in Q with constant T_{in}
- Compare step response with the linear model

Response comparison

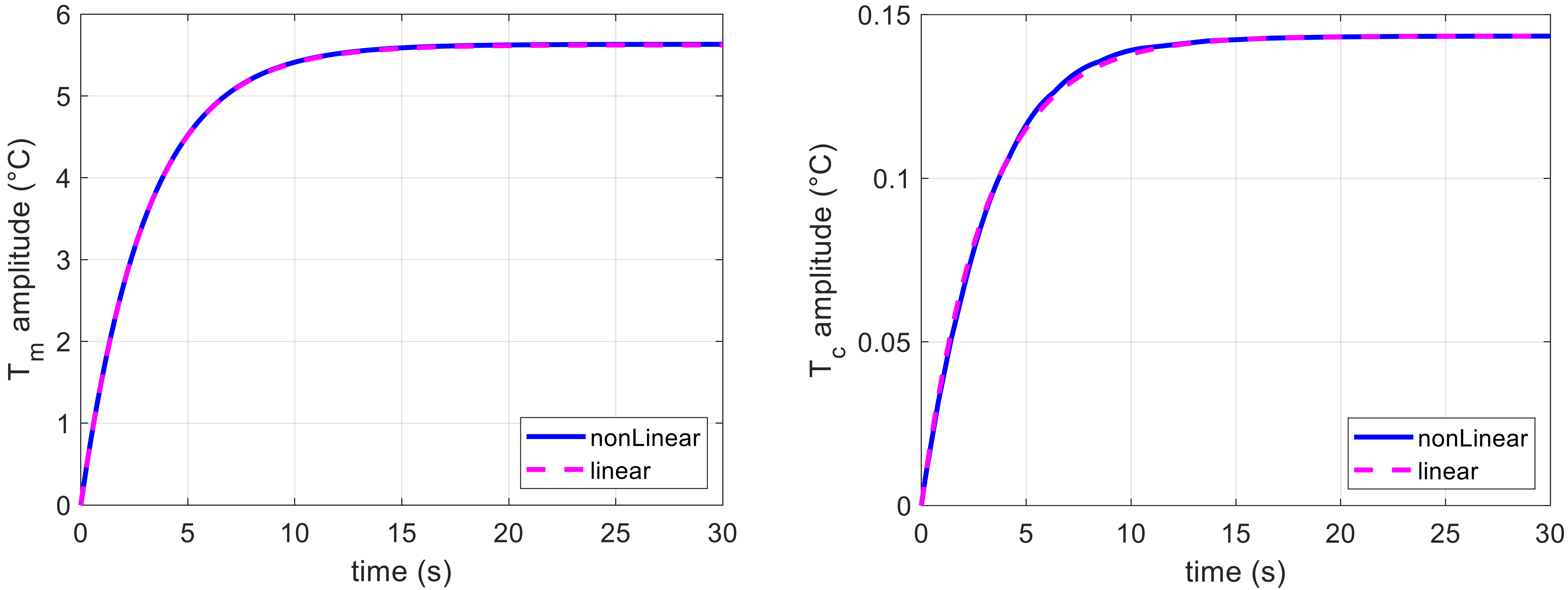
Perturbation of 1 °C of T_{in} (constant Q)



- Both T_m and T_c presents very similar behaviour in the two solutions!
- The stady state is reached within similar time and the amplitude is comparable

Response comparison

Perturbation of 1 kW of Q (constant T_{in})



- Both T_m and T_c presents very similar behaviour in the two solutions
- The stady state is reached within similar time and the amplitude is comparable

Conclusions and future perspectives

- A linear model accounting for linear expansion is devolped
- The stability of the configuration is verified through the Liapunov’s criterion
- The linear model is verified through a comparision of step responses with the non linear solution

- Expand the model to 1D
- Include electro-magentic physics in the model
- Use the model for control

Back up slides

Numerical model and results

- Mesh deformation mapping the displacement from the thermo-mechanical simulation
- → Thermal-hydraulic simulation to check the temperature

New peak temperature = 202 °C < 219 °C

→Lower displacement